

# Theoretical Derivation of the Schwarzschild Radius using Newton's Escape Velocity:

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## Abstract

Black holes are among the most fascinating objects in astrophysics because their gravitational pull is so strong that not even light can escape. The boundary surrounding a black hole is called the event horizon, and its radius is known as the Schwarzschild radius. This paper presents a simple theoretical derivation of the Schwarzschild radius using basic Newtonian gravitational principles and the concept of escape velocity. By equating the escape velocity of an object to the speed of light, the same mathematical form of the Schwarzschild radius can be obtained as predicted by general relativity. This approach provides an intuitive bridge between high-school level physics and modern astrophysics, helping students understand how fundamental physical principles can approximate complex relativistic results for a non-rotating and spherically symmetric mass distribution.

## 1. Introduction

Black holes have long fascinated physicists and students alike because they represent a limit where gravity dominates all other forces and even light cannot escape. When a sufficiently massive star exhausts its nuclear fuel, it can undergo gravitational collapse and form a black hole, surrounded by a boundary from which nothing can escape, known as the event horizon; for a non-rotating, uncharged black hole, this boundary is characterized by the Schwarzschild radius. Karl Schwarzschild first obtained this radius in 1916 as an exact solution of Einstein's field equations of general relativity, just months after Einstein published them, and the idea of such extremely compact objects became widely accepted thereafter. Interestingly, a similar expression for this radius can be obtained using only Newtonian gravity and the classical notion of escape velocity, which makes the concept accessible at the high school level and provides strong physical intuition.

This Newtonian approach builds understanding by showing how, as an object's size shrinks while its mass stays fixed, the escape speed from its surface increases until it matches the speed of light, at which point even light cannot escape and an event horizon forms. By applying the concept of escape velocity and equating it to the speed of light, we can estimate the size of this boundary using only basic Class 11 physics formulas like Newton's law of gravity and the standard escape velocity equation. Remarkably, this leads to the same mathematical form as the Schwarzschild radius derived from general relativity, offering a powerful bridge between school-level physics and advanced modern astrophysics. This paper presents that step-by-step derivation and explains the physical meaning behind it in a way that is accessible to students.

## 2. Formation of Black Holes

Before the derivation, let's understand the formation of Black holes. Black holes are generally formed from the gravitational collapse of very massive stars. When a massive star exhausts its nuclear fuel, the outward pressure produced by fusion decreases. Gravity then causes the star to collapse inward. If the mass of the remaining core is sufficiently large, the collapse continues until an extremely dense object is formed where the gravitational pull becomes extraordinarily strong. At a certain radius, the escape

velocity becomes equal to the speed of light, creating the event horizon. Beyond this boundary, nothing can return to the outside universe.

### 3. Escape Velocity and Gravitational Energy

Escape velocity is the minimum velocity required for an object to escape the gravitational field of a massive body without further propulsion. It can be derived using conservation of energy. We know that, energy of an object is the sum of potential and kinetic energy. Since it is under gravitational field we can take potential as gravitational energy.

Then,

$$\text{Kinetic energy} + \text{Gravitational Potential Energy} = 0 *$$

$$\text{Kinetic Energy} = \frac{1}{2} m v_e^2$$

$$\text{Gravitational Potential Energy} = - \frac{Gm}{r_s} M_e$$

On equating them, we get

$$\frac{1}{2} m v_e^2 = \frac{Gm}{r_s} M_e$$

Where,

$m$  = mass of the object

$M_e$  = mass of the celestial body

$r_s$  = distance from the center

$G$  = gravitational constant

Solving for velocity gives the escape velocity formula,

$$v_e = \frac{\sqrt{2GM_e}}{r_s}$$

This formula shows that stronger gravity or smaller radius increases the escape velocity.

### 4. Derivation of the Schwarzschild Radius

For a black hole, the escape velocity at the event horizon equals the speed of light  $c$ . By substituting  $v_e = c$  in the escape velocity equation we obtain,

$$c = \frac{\sqrt{2GM_e}}{r_s}$$

\*- Escape velocity uses conservation of energy: initial kinetic energy exactly cancels the negative gravitational potential energy. At infinite distance, gravitational pull vanishes, making potential energy zero. The object arrives there with zero kinetic energy too—no leftover.

Squaring both sides gives,

$$c^2 = \frac{2GM_e}{r_s}$$

This radius is known as the Schwarzschild radius. It represents the boundary beyond which nothing can escape the gravitational pull of the black hole. Although derived using classical mechanics, this result matches the exact expression obtained from general relativity for a non-rotating spherical mass.

## 5. Physical Interpretation

The Schwarzschild radius helps us understand how mass affects the formation of black holes. If any object were compressed within its Schwarzschild radius, it would theoretically become a black hole. For example, if the Sun were compressed into a sphere with a radius of about 3 km, it would become a black hole. This concept highlights the extreme density required for such objects to form.

## Conclusion

This paper demonstrates that the Schwarzschild radius, which defines the event horizon of a black hole, can be derived using simple Newtonian physics and the concept of escape velocity. By equating escape velocity with the speed of light, the same mathematical expression predicted by general relativity can be obtained. Although this approach does not fully describe the complex nature of black holes, it provides an accessible understanding of their basic properties. The derivation also illustrates how fundamental physics concepts learned in school can connect to advanced topics in astrophysics.

## References

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